Cultural propagation on social networks

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In this work we present a model for the propagation of culture on networks of different topology and by considering different underlying dynamics. We extend a previous model proposed by Axelrod by letting a majority govern the dynamics of changes. This in turn allows us to define a Lyapunov functional for the system.

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INTRODUCTION

Over the last few years it has been possible to witness the increasing interest in mathematical models aimed to describe and analyze social processes. A fruitful symbiosis took place, establishing collaborations among researchers from the social sciences and physics as well as mathematics. The process gave birth to an interesting collection of works dealing with a wide spectra of social phenomena thus analyzed through a variety of mathematical and physical techniques [1–3].

Some important and primordial questions constitute the motivation to work on these models. First, it is important to know how the behavior of unorganized individuals within a society contributes to produce collective social phenomena. Next, it is necessary to know how stable this emergent phenomena are. Another important aspect is knowing to what level social organization is encoded within the topology of the system, or what is the extent of the effect of the social structure on the particular characteristics of the evolution of a given social scenario. A preponderant role in relation with this aspect has been played by works on social or complex networks.

Proposing different social networks as schemes for the underlying architecture of the society, many authors have presented models to describe opinion formation [1,4], rumors [5], diseases [6,7], fashion propagation [8,9], urban segregation [10], majority vote [11], etc. In this work we will analyze a generalization of a model of culture propagation introduced by Axelrod [3,12]. In the original model, the cultural background of an individual is characterized by a set of F dynamical attributes or cultural features that evolve according interactions with the social environment. Each feature, in turn, can take q different values, representing possible traits. The individuals are located on top of a regular network and interact with their neighbors. Through this interaction the cultural profile of each individual, and thus the configuration of the system, evolve. The interaction is mediated by what is called cultural affinity. The more similar an individual is to one of its neighbors, the more likely the interaction is. The later consists in the adoption of a common trait in one of the F cultural aspects. Typically, the system evolves towards a monocultural state, but for some parameter values it freezes in a multicultural state with coexisting spatial domains of different cultures. The number of these domains is taken as a measure of cultural diversity.

A systematic analysis of the dependence on q of the original model was carried out in Ref. [13]. Further analysis of the model and of the role of noise was perform in Refs. [14,15]. In this work we have modified the original formulation, allowing for a wider and more thorough evaluation of the cultural environment that surrounds each individual and thus influences on its cultural tendencies. Similar considerations have been made in Refs. [16,17]. Each individual will evaluate whether or not to copy one of the traits adopted by one of its neighbors, basing the decision on an observation of the state of its entire neighborhood. A sort of majority rule will govern the dynamics of the system. We propose different forms for this majority rule. In each case, a function of the state of the system that behaves monotonically in time is found. This function can be associated with a Lyapunov function of the system. The idea of defining a Lyapunov potential in the Axelrod's model was recently reviewed in Ref. [18]. The monocultural state is always the absolute minimum of this function, though there exist some local minima, corresponding to multicultural situations where the system remains frozen. The proposed scheme of interaction among individuals mimics a social situation in a way that approaches more to the real interactions within the cultural broth. At the same time, the introduced changes allow the system to reach a situation of nonfrozen macroscopical steady state. The system is dynamical, the cultural profiles of the individuals evolve and change only driven by their interactions, without the addition of external noise, while the system preserves some macroscopic properties such as the degree of multiculturality. Other interested models on cultural evolution have been presented in Refs. [19,20]. The mentioned works analyze under what circumstances the cultural diversity arises and survives despite the homogenizing interaction. In Ref. [21], the authors are rather interested in the emergence of a coherent culture in an heterogeneous interacting population.

THE MODEL

Our work is based on a previous model by Axelrod [3,12]. We consider that the cultural background of any individual can be characterized, in a quite reductionist way, by a given set of nonoverlapping features. We will call F the number of these features ϕ_k that define the culture in a schematic way. In principle, each feature can be associated to a different aspect of the culture such as spoken language, preferred foods, or musical style, readings, sports, etc. In turn, a further subclassification of each feature into categories will serve to denote the different preferences or traits. The simplest consideration to achieve such subdivision is to consider that any cultural feature may take on any of q different values, the same for any ϕ_k. In principle the values are only labels, so despite the fact that we use numbers for the classification, any set of symbols would work as well. Thus, an individual i is culturally characterized by a cultural vector of F components ϕ_k^i, each one adopting values ranging between 1 and q. The way to culturally compare two individuals i and j is to measure their cultural overlap, $\omega_{(i,j)}$ as follows:

$$\omega_{(i,j)} = \sum_{k=1}^{F} \delta_{\phi_k^i \phi_k^j}. \tag{1}$$

The individuals are situated on the vertices of a graph. Two of them are neighbors when linked by an edge. In the original model, the underlying network was a bidimensional regular lattice [12]. The topology of the network was later generalized in [14], considering amongst others, small world (SW) networks [2]. Here, we also consider SW networks.

As the individuals interact with the set of their neighbors, it is useful to define at this point the local overlap of a given individual Ω_i as

$$\Omega_i = \sum_{j \in \sigma_i} \omega_{(i,j)}, \tag{2}$$

where σ_i is the set of neighbors of i. We will also define the quadratic local overlap as

$$\Theta_i = \sum_{j \in \sigma_i} \omega_{(i,j)}^2. \tag{3}$$

Starting from an initial distribution of cultural vectors, the individuals evolve by analyzing and interacting with their environment, adapting their cultural preferences according to the tendencies of the neighborhood. The original numerical simulations proceed as follows in Refs. [12,14,15]. At time step t, a randomly chosen individual i and one of its neighbors j are evaluated. Their cultural overlap is calculated to decide whether they will interact or not. The interaction takes place with a probability $\omega_{(i,j)}(t)/F$, in which case one of the features ϕ_k^i such that $\phi_k^i \neq \phi_k^j$ is set equal to ϕ_k^j. Though it is evident that $\omega_{(i,j)}(t) \geqslant \omega_{(i,j)}(t-1)$, the interaction may affect as well the overlaps between i and the rest of the neighborhood and thus the change on Ω_i cannot be anticipated.

Interesting results were obtained in Ref. [15] by considering the whole process of cultural dissemination as an optimization problem. In that work, by analyzing a one dimensional system with interaction amongst the first neighbors, the authors found a Lyapúnov potential that allowed them to analyze the stability of the states at which the system remained frozen after some evolution time. We are interested in extending those results to more general situations, namely, SW and other complex networks. The global overlap

$$\Omega = \frac{1}{2} \sum_i \Omega_i \tag{4}$$

cannot be claimed to have a monotonic behavior when the system, as defined, evolves in time. Suppose that as a result of the interaction between i and j at time t there is a change in the value of ϕ_k^i. We will call σ_i^m the neighborhood of i such that for any of the μ individuals $h_m \in \sigma_i^m$, $\phi_k^{h_m}(t) = \phi_k^i(t)$; and σ_i^n the set of ν neighbors h_n such that $\phi_k^{h_n}(t) = \phi_k^i(t+1)$. The rest of the neighbors will be included in the set σ_i^l. If at each time step only one change is allowed, we can calculate $\Delta\Omega = \Omega(t+1) - \Omega(t) = [\Omega_i(t+1) - \Omega_i(t)]$ by considering that

$$\begin{aligned}\Omega_i(t+1) &= \sum_{j \in \sigma_i^l} [\omega_{(i,j)}(t)] + \sum_{j \in \sigma_i^m} [\omega_{(i,j)}(t) - 1] \\ &\quad + \sum_{j \in \sigma_i^n} [\omega_{(i,j)}(t) + 1] \\ &= \sum_{j \in \sigma_i} [\omega_{(i,j)}(t)] - \sum_{j \in \sigma_i^m} 1 + \sum_{j \in \sigma_i^n} 1.\end{aligned} \tag{5}$$

Thus

$$\Omega_i(t+1) = \Omega_i(t) + \nu - \mu. \tag{6}$$

The change in Ω is thus $\Delta\Omega = \nu - \mu$, which is not necessarily equal or greater than zero. By introducing a modification in the dynamics of the original model we can assure that this condition will be fulfilled and thus can think of a Lyapunov functional for the system.

We will consider different types of dynamics, each one associated with a corresponding Lyapunov function but at the same time with a clear social interpretation of the behavior of the individuals. The underlying network will be built up following the procedure described in Ref. [22]. In the original model of SW networks, a single parameter p, running from 0 to 1, characterizes the degree of disorder of the network, respectively, ranging from a regular lattice to a completely random graph. The construction of these networks starts from a regular, one-dimensional, periodic lattice of N elements linked to $2K$ nearest neighbors. Then each of the sites is visited, rewiring K of its links with probability p. Values of p within the interval $[0,1]$ produce a continuous spectrum of small world networks.

CULTURAL EXCHANGE DYNAMICS

Case 1: Restricted cultural affinity

The original model proposed by Axelrod considered a very special case of biased dynamics for the interactions of the individuals. Despite the fact that individuals are immersed in their neighborhood, this was ignored by requiring that the individual interact with only one of its neighbors. Taking this fact into account, a first adaptation of the original model consists in deciding whether to change or not the value of the chosen feature by weighting the decision with a further evaluation of the influence of the neighborhood. Given that the individual i interacts with j, the possibility of adopting ϕ_k^j for ϕ_k^i will depend on the result of an evaluation following a sort of majority rule. If by accepting the change of the value of ϕ_k^i i will share the value taken by ϕ_k^i with a bigger group than if by rejecting the change, then i accepts the change. With a probability 1/2 the change is accepted in case of equality. This is translated into the following situation. The change is accepted whenever $\nu > \mu$, and with probability 1/2 when $\nu = \mu$. Under this condition, $\Omega(t+1) - \Omega(t) \geqslant 0$. So, we will take $\pounds_1(t) = -\Omega(t)$ as the Lyapunov function of this dynamics, and the system will evolve to reach a local or absolute minimum.

Case 2: Complete cultural affinity

The former rule assures that the global cultural background grows or at least is maintained constant, while the individuals knows that accepting the change will warrant being in a bigger group regarding the changing feature. But basing the decision of the individual on the comparison of only one feature seems quite myopic. We can propose another condition, making the individual base the decision on a further evaluation of the local partial overlaps $\Omega_i^m = \sum_{j \in \sigma_i^m} \omega_{(i,j)}$ and $\Omega_i^n = \sum_{j \in \sigma_i^n} \omega_{(i,j)}$. Despite that one feature was chosen to be changed, the individual decides whether to adopt the new value or not by weighting the whole cultural overlap with its neighborhood and not by analyzing what happens with the specific feature to be changed. This is equivalent to saying that the individual will favor a majority weighted by deeper cultural affinity. Now, we can no longer say that $\Omega(t+1) - \Omega(t) \geqslant 0$. We must look for another quantity $\pounds_2(t)$, such that $\pounds_2(t+1) - \pounds_2(t) \leqslant 0$. In what follows we show that $\pounds(t)_2 = -[\Theta(t) + \Omega(t)]$, with $\Theta(t) = \frac{1}{2}\sum_i \Theta_i(t)$, satisfies the required condition.

Let us consider that in the proposed interaction between i and j, ϕ_k^i will be change by ϕ_k^j. There are three classes of individuals among the neighbors of i, those belonging to σ_i^m, those belonging to σ_i^n and the rest, that will be grouped in σ_i^l. The local partial overlap $\Omega_i^l = \sum_{j \in \sigma_i^l} \omega_{(i,j)}$ will no be affected, regardless of whether or not the interaction takes place. On the contrary, if the interaction occurs at time t, $\Omega_i^m(t+1) = \Omega_i^m(t) - \mu$, and $\Omega_i^n(t+1) = \Omega_i^n(t) + \nu$.

If no interaction is allowed, $\pounds_2(t+1) = \pounds_2(t)$. If on the contrary, the change is accepted, we have

$$\pounds_2(t+1) - \pounds_2(t) = [\Theta_i(t+1) - \Theta_i(t)] + [\Omega_i(t+1) - \Omega_i(t)].$$

We can expand the right-hand side of the former equation by considering sums over σ_i^m, σ_i^n, and σ_i^l. On one side we have

$$\begin{aligned}\Theta_i(t+1) &= \sum_{j \in \sigma_i^l} [\omega_{(i,j)}(t)]^2 + \sum_{j \in \sigma_i^m} [\omega_{(i,j)}(t) - 1]^2 \\ &\quad + \sum_{j \in \sigma_i^n} [\omega_{(i,j)}(t) + 1]^2 \\ &= \sum_{j \in \sigma_i} [\omega_{(i,j)}(t)]^2 + \sum_{j \in \sigma_i^m} 1 - 2\omega_{(i,j)}(t) \\ &\quad + \sum_{j \in \sigma_i^n} 1 + 2\omega_{(i,j)}(t). \end{aligned} \tag{7}$$

Expanding and regrouping terms we get

$$\begin{aligned}\Theta_i(t+1) &= \Theta_i(t) + \Omega_i^n(t+1) - \Omega_i^m(t+1) + \Omega_i^n(t) - \Omega_i^m(t), \\ &= \Theta_i(t) + 2[\Omega_i^n(t+1) - \Omega_i^m(t)] + \mu - \nu.\end{aligned}$$

On the other hand, we have Eq. (2). Finally

$$\pounds_2(t+1) - \pounds_2(t) = -\{2[\Omega_i^n(t+1) - \Omega_i^m(t)]\}.$$

The condition to be fulfilled is

$$\Omega_i^n(t+1) - \Omega_i^m(t) \geqslant 0$$

that corresponds to the imposed constraint.

It is important to note that in all the cases, the monocultural state corresponds to minimum value of the Lyapunov function $\pounds_i^M$. We can use this value for normalization, such that $L_i = \pounds_i / \pounds_i^M$.

NUMERICAL RESULTS

In what follows we will include results corresponding to the cases 1 and 2 as well as those corresponding to the Axelrod' s original model, for which we have not defined a Lyapunov function. We have performed extensive numerical simulations of the described model, considering different dynamics. The networks have $N = 10^4$ vertices and connectivity $K = 2$. A typical realization starts with the generation of the random network and the initialization of the state of the elements. After a transient period, the duration of which depends on the parameters of the particular simulation, a macroscopic stationary state is achieved. The computations are then repeated for several thousand time steps to perform statistical averages. We consider that the system has achieved a macroscopic stationary state when the corresponding Lyapunov function of the systems reaches a stationary value. We will see that this does not imply that the system is steady in a particular microscopical state. Indeed, the configuration of the system fluctuates among states associated to equal values of the Lyapunov function. There are several aspects characterizing the asymptotic evolution of the system to a stationary value of the Lyapunov function. We also analyze the behavior of the system when governed by the original dynamics, in which case the steady state is not characterized by a Lyapunov function and achieves a microscopically frozen state.

In all the calculations, we took $F = 10$ and several values of q, ranging from 2 to 80. At each time step only one change was allowed, the system was updated asynchronically. We considered that one unity of time corresponded to N time steps.

For each of the dynamics described above, we have analyzed several aspects of the evolution of the system. First we have calculated the proportion of overlaps ρ between individuals corresponding to three cases, (a) ρ_0 when $\omega_{(i,j)} = 0$, (b) ρ_F when $\omega_{(i,j)} = F$, and (c) ρ_a when $0 < \omega_{(i,j)} < F$. Cases (a) and (b) correspond to the situation when no change in the system is possible because the interaction of two individuals: in case (a) because no interaction will occur when the cultural overlap is equal to zero, in the case (b) because individuals are already culturally identical. The only active links

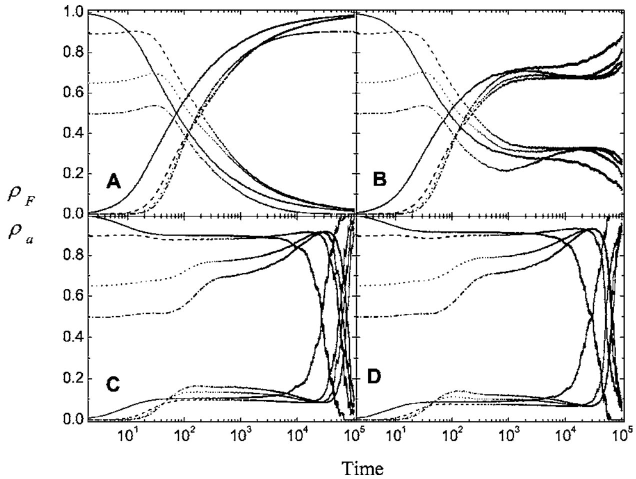

FIG. 1. Proportion of active links ρ_a and of complete overlap links ρ_F vs time, with $q=2$ (full), $q=5$ (dashed), $q=10$ (dotted), $q=15$ (dotted-dashed). Each plot correspond to a different value of p: (A) $p=0$, (B) $p=0.01$, (C) $p=0.5$, (D) $p=0.9$. Axelrod's case.

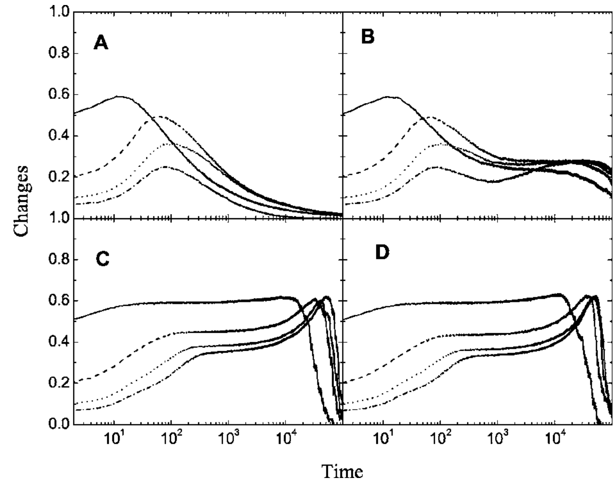

FIG. 2. Proportion of changes vs time, for different values of q and p as in Fig. 1. Axelrod's case.

are those corresponding to case (c). Then we have calculated the corresponding Lyapunov function (when defined) to show how its value evolves monotonically to a steady one. Though this does not provide any information about the inner structure of the system, or about the existence of clusters, it helps us to have an idea of the amount of cultural differentiation that is present. To show that though the Lyapunov function reaches a steady value, but the system is not in a steady state, we calculated the amount of changes that occur in each time step. This was also useful to show that in the Axelrod case, the system attained a frozen state, with no changes.

Axelrod's case

This case corresponds to the original model [12,14,15] where individuals interact with only one of their neighbors at each time step. The interaction is mediated by the cultural affinity, defined through the cultural overlap $\omega_{(i,j)}$. The stronger the affinity is, the greater the possibility of interaction between two subjects. In the present work, the individuals are located on networks with different degrees of disorder. The ordered case $p=0$, corresponds to a one-dimensional lattice with interactions between the first and second neighbors.

As stated before, we did not find a Lyapunov function for this case, and we restrict the displayed results to the time dependence of the proportion of overlaps ρ_F and ρ_a, and of the proportion of changes in the individuals' cultural profiles. We recall that $\rho_0=1-(\rho_F+\rho_a)$. Figure 1 displays the time evolution, averaged over 1000 realizations, of the values ρ_F and ρ_a, corresponding to the amount of overlaps $\omega_{(i,j)}=F$ and $0<\omega_{(i,j)}<F$, normalized to the total number of links KN. We evaluate these quantities on networks with different degree of disorder, namely, $p=0,0.01,0.5,0.9$.

A deeper insight into what is happening is obtained by analyzing the data contained in Fig. 2. There we show the proportion of changes in the cultural vectors of the individuals. Each change corresponds to a component of any cultural vector that changed its value. We show the amount of changes in a unit of time normalized to the maximum value allowed N, the number of proposed changes.

Figures 1(a) and 2(a) correspond to an ordered underlying network. The system goes to a state where only non active links survive, that is, $\rho_a\to 0$. At the same time, while the system reaches a steady state, associated with the number of changes approaching 0, the system achieves a monocultural state when $q\leq F$ but goes to a multicultural state for higher values of q.

When some disorder is introduced into the network, the behavior of the system is more complex. By looking at Fig. 2 we can see that there are two different behaviors for ordered and very disordered networks while the intermediate case, $p=0.01$ shows a mixture of both. The system, in ordered networks evolves rather fast to a state of low multiculturality or monoculturality. When the disorder is increased, the initial disorder survives for longer times. At the end, the system ends in a monocultural state except when $p=0$ and $q>F$. Figure 1, showing the number of changes in time confirms what was mentioned before. We have not observed sharp transitions while varying the p value, indeed, the behavior of the system undergoes a smooth change as the spatial disorder is increased.

Case 1: Restricted cultural affinity

In the following cases the calculation of the Lyapunov function will provide us additional information about the system behavior. As in the previous case, Fig. 3 displays the time evolution, averaged over 1000 realizations, of the values ρ_F and ρ_a, evaluating these quantities on networks of varying disorder. Figure 5 shows the evolution in time of the proportion of changes. The evolution of the normalized Lyapunov function corresponding to this case, L_1 is plotted in Fig. 4. Starting from the ordered case, Fig. 3(a), we observe that results do not differ so much from what we have previously seen. Again, the system goes to a state where only non active links survive, reaching a steady state, and achiev-

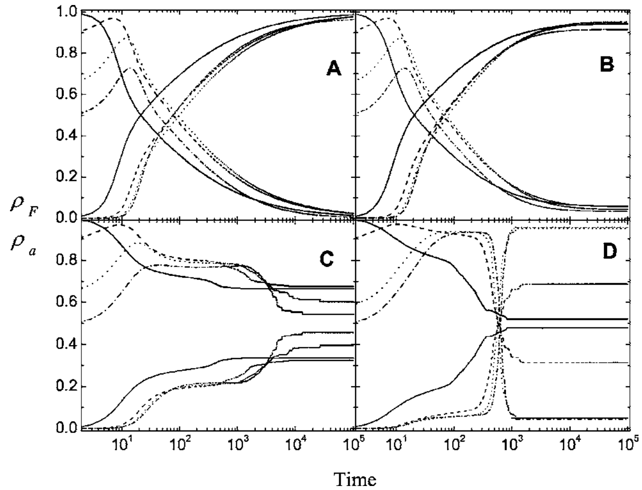

FIG. 3. Proportion of active links ρ_a and of complete overlap links ρ_F vs time, for different values of q and p as in Fig. 1. Case 1.

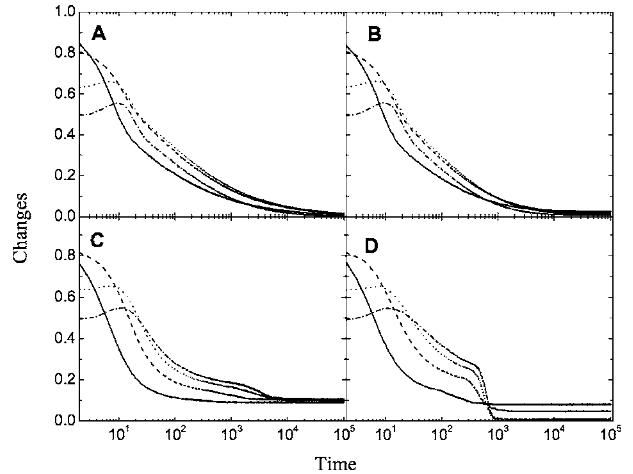

FIG. 5. Proportion of changes vs time, for different values of q and p as in Fig. 1. Case 1.

ing monoculturality when $q \leqslant F$ and a certain degree of multiculturality for higher values of q.

This time we can recur to the Lyapunov function to see that the absolute minimum is reached when $q \leqslant F$, but the system remains in a frozen state of multiculturality when $q > F$. It is interesting to observe that the number of changes, Fig. 5(a), goes to zero.

When disorder is included on the network, the behavior of the system displays nontrivial effects as can be can observed in Figs. 3(b)–3(d). The number of active links is different from zero, even when a steady value for the Lyapunov function is reached. Though the monocultural state is the absolute minimum, it is note attained by the system, who finishes trapped in local minimum. In Figs. 4(b)–4(d) we observe that the Lyapunov function decreases monotonically to attain a steady state but not to the absolute minimum. On the other hand, the steady values depend non monotonically on the disorder of the network. Despite the fact that $\pounds_1$ remains steady, the state of the system is not frozen. This affirmation comes from the observation of Fig. 5, where we find that the number of changes remains above zero in all cases. Again, the mean value of changes behaves in a nontrivial way when q or p change.

Perhaps the most interesting feature is the interplay between the effect of the spatial disorder and the values of q. This can be better observed by analyzing the behavior of the Lyapunov function. In some cases the disorder introduced by the network prevents the system from achieving the previously reached monocultural state, but on the other hand, the final degree of multiculturality depends in a very interesting way from both parameters. An interesting nonmonotonic behavior of L_1 in terms of p can be observed depicted in Fig. 6.

Case 2: Complete cultural affinity

The first aspect that we can observe for this case is that independently of the degree of disorder of the network, the state of monoculturality is never achieved, as shown in Fig. 7. We can again verify the interplay between the parameters q and p and their effect on the behavior of the system. An-

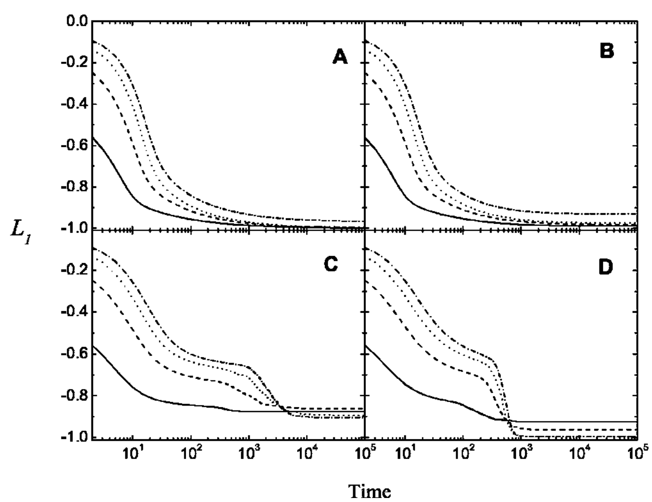

FIG. 4. Normalized Lyapunov function L_1 vs time, for different values of q and p as in Fig. 1.

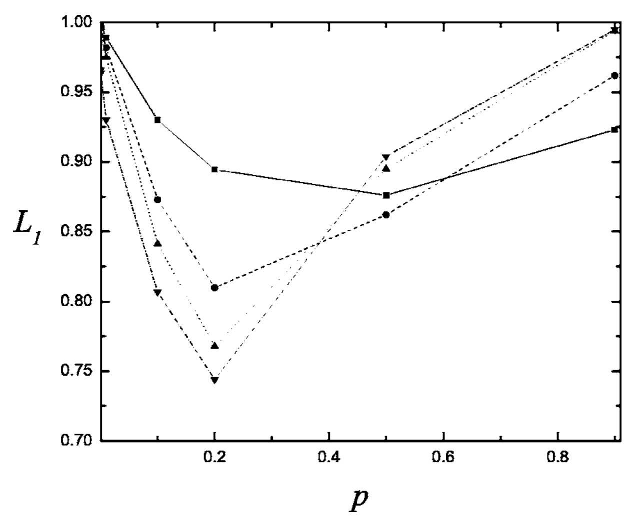

FIG. 6. Steady value of $-L_1$ vs p, with $q=2$ (full), $q=5$ (dashed), $q=10$ (dotted), $q=15$ (dotted-dashed).

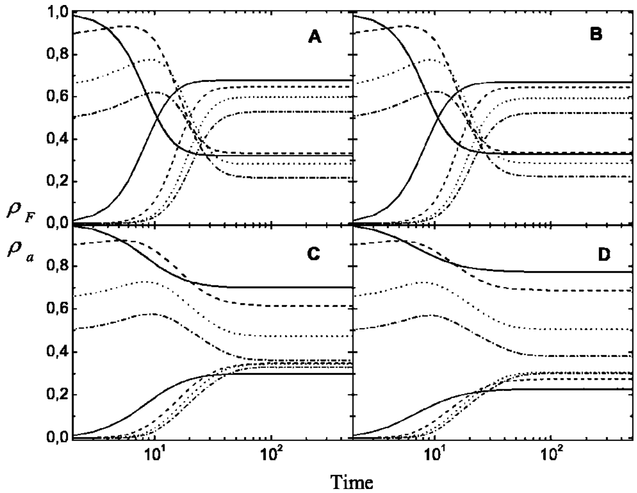

FIG. 7. Proportion of active links ρ_a and of complete overlap links ρ_F vs time, for different values of q and p as in Fig. 1. Case 3.

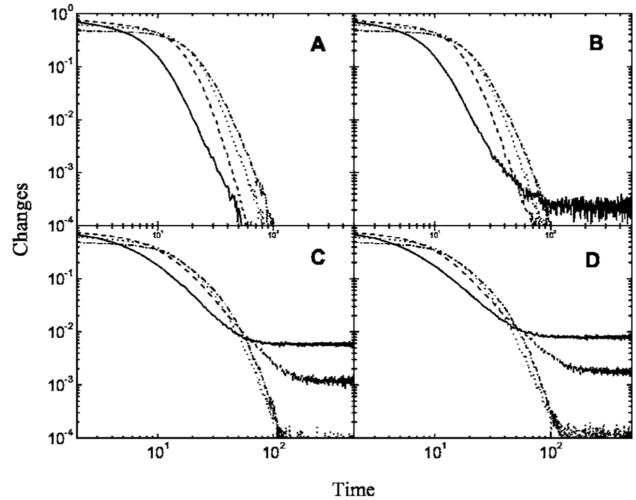

FIG. 9. Proportion of changes vs time, for different values of q and p as in Fig. 1. Case 2.

other issue to be observed is the time scale. The evolution towards a steady value of the Lyapunov function is much faster than before, as observed in Fig. 8. At the same time we observe that by increasing the disorder the amount of active links grows.

This is associated to the fact observed in Fig. 9, where we can see how the amount of changes in the final state also increases with p. The behavior of the Lyapunov function simply verifies that the system reaches a steady value and that this value is far from being the absolute minimum. In all the cases, the steady value decreases with q.

CONCLUSIONS

Axelrod's model shows how a microscopical local process of interaction, leading to convergence provokes the emergence of global polarization. In previous works, the model was used to analyze the effect of the number of cultural aspects and traits on the steady configuration of the system. Further analysis [13] of the relative size of the largest cultural domain revealed an order disorder transition with q, the number of different traits, playing the role of the control parameter. Under a threshold value $q_c(F)$, the system converges to a monocultural uniform state. Above $q_c(F)$ the system freezes in a multicultural state, that can be associated to polarization. The stability of the multicultural states was analyzed in Ref. [14] by perturbing the system when frozen in a multicultural state and showing the further convergence to the monocultural state. Perturbations were associated to cultural drift.

In this work we proposed a different sort of generalization of Axelrod' s model. We modified the model to include interactions among several individuals within a neighborhood

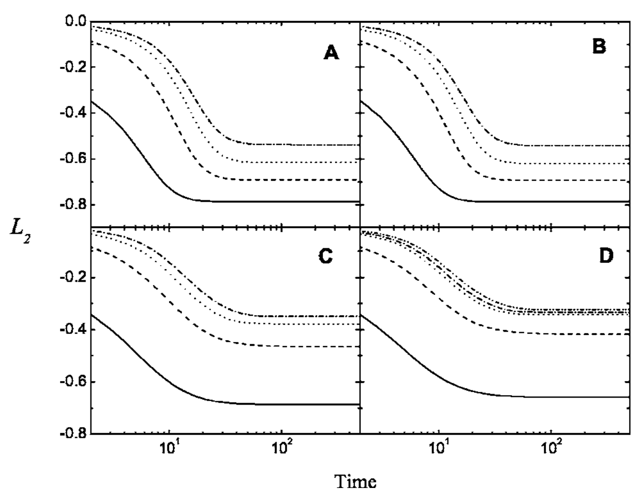

FIG. 8. Normalized Lyapunov function L_2 vs time, for different values of q and p as in Fig. 1.

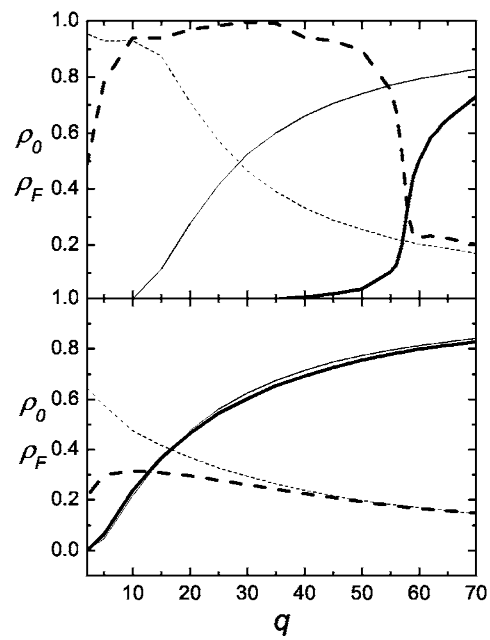

FIG. 10. Asymptotic proportion of inactive links ρ_0 (solid) and ρ_F (dashed) for different values of q. With bold line: $p=0.9$ and thin line $p=0$, solid line. (a) Case 1, (b) case 2.

or to let each individual evaluate the changes in its cultural preferences by analyzing those of its neighbors. The cultural influence of the environment was already studied in Ref. [16].

Different ways of considering this extended interaction were shown. For each, an associated Lyapunov function was found, letting us analyze the convergence of the system towards an absolute or local minima. The disorder of the system was not reduced to that introduced by the initial condition by increasing the value of q, but also included in the spatial distribution of the agents. For this purpose we analyzed the effect of the disorder of the underlying network considering small world networks of varying disorder. The results linked to this aspects can be compared with previous results and thus unveil the effect of the newly defined interaction of each individual with the whole neighborhood. As already known, increasing the value of q leads the system to undergo a transition from monoculturality to multiculturality. However, when the dynamics of the system corresponds to the case 1, the effect of spatial disorder attempts against this effect. Figure 10 shows the asymptotic values of ρ_0, ρ_a, and ρ_F for different values of q and dynamics. Figure 10(a) corresponding to the case 1 and Fig. 10(b) to the case 2. In Fig. 10(a) it is possible to observe the transition from monoculturality to multiculturality at different values of q. This can be explain by recalling that in a disordered network the clusterization of the system is lower and thus, the existence of clusters of culture reflected in a polarized situation is no longer achieved. When the slightest disorder is added to the network, the number of links with overlap equal to zero decays. In case 2, the transition to the multicultural state occurs at lower values of q when compared with previous results. It is important to recall that multiculturality presents here a different character. The change of the rules of interaction introduces a new interesting behavior. Not only does the amount of active links not go to zero, with the exception when the underlying lattice is ordered and $q=2$, but also the system reaches a situation when the Lyapunov function adopts a steady value but the system is not frozen. The configuration of the system changes in time, as can be observed from the figures displaying the number of changes in time.

The results presented here complement what was already found in the analysis of the model first proposed by Axelrod. The interesting feature is that the system, despite reaching a steady situation, does not remains static. Some aspects still deserve further analysis. Among them we will consider in a future work the inclusion of noise.

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